Find the value of $\sum_{n=2}^{\infty} 600(0.5)^{4n-6}$. **HINT: Write out the first few terms first.** **SCORE: _____ / 15 PTS**

$= 600(0.5)^2 + 600(0.5)^6 + 600(0.5)^{10} + \cdots$

INFINITE GEOMETRIC SERIES

$r = (0.5)^4 = \frac{1}{16}$

3

$= \dfrac{600(\frac{1}{2})^2}{1 - \frac{1}{16}}$ 4

4

$= \dfrac{150}{\frac{15}{16}} = \cancel{150}^{10} \cdot \dfrac{16}{\cancel{15}} = 160$

4

Consider the sequence defined recursively by $a_n = \frac{7}{3}a_{n-1} + 2a_{n-2}$, $a_1 = -\frac{9}{8}$, $a_2 = \frac{3}{4}$. SCORE: _____ / 20 PTS

[a] Find the first 6 terms of the sequence. **Your answers must be integers or fractions, NOT decimal approximations.**

$a_1 = -\frac{9}{8}$, $a_2 = \frac{3}{4}$

$a_3 = \frac{7}{3}a_2 + 2a_1 = \frac{7}{\cancel{3}}\left(\frac{\cancel{3}}{4}\right) + \cancel{2}\left(-\frac{9}{\cancel{8}_4}\right) = \frac{7}{4} - \frac{9}{4} = -\frac{1}{2}$, $3\frac{1}{2}$

$a_4 = \frac{7}{3}a_3 + 2a_2 = \frac{7}{3}\left(-\frac{1}{2}\right) + \cancel{2}\left(\frac{3}{\cancel{4}_2}\right) = -\frac{7}{6} + \frac{3}{2} = -\frac{7}{6} + \frac{9}{6} = \frac{1}{3}$, $3\frac{1}{2}$

$a_5 = \frac{7}{3}a_4 + 2a_3 = \frac{7}{3}\left(\frac{1}{3}\right) + 2\left(-\frac{1}{2}\right) = \frac{7}{9} - 1 = \frac{-2}{9}$, $3\frac{1}{2}$

$a_6 = \frac{7}{3}a_5 + 2a_4 = \frac{7}{3}\left(-\frac{2}{9}\right) + 2\left(\frac{1}{3}\right) = -\frac{14}{27} + \frac{2}{3} = \frac{-14}{27} + \frac{18}{27} = \frac{4}{27}$, $3\frac{1}{2}$

[b] Based on the first 6 terms, does the sequence appear to be arithmetic, geometric or neither ? Show how you reached your conclusion.

GEOMETRIC, 2

$\frac{3}{4} / -\frac{9}{8} = \frac{\cancel{3}}{\cancel{4}} \cdot \frac{-\cancel{8}^2}{\cancel{9}_3} = -\frac{2}{3}$

$\frac{\cancel{3}}{\cancel{4}_2} \cdot \frac{-\cancel{2}}{\cancel{3}} = -\frac{1}{2}$

$-\frac{1}{2} \cdot -\frac{2}{3} = \frac{1}{3}$

$\frac{1}{3} \cdot -\frac{2}{3} = \frac{-2}{9}$

$-\frac{2}{9} \cdot -\frac{2}{3} = \frac{4}{27}$

4

QJ got a new credit card on December 1, 2015, and charged \$47 on it that day. On the 1st day of every month after that, QJ charged 1.3% more than she had charged on the 1st day of the previous month. By December 1, 2017, how much had QJ charged on her card altogether ? (Assume that QJ never charged anything else to her card except on the 1st day of each month.)

SCORE: _____ / 15 PTS

$$47 + 47(1.013) + 47(1.013)^2 + \dots + 47(1.013)^{25-1}$$ GEOMETRIC SERIES

$$= \boxed{\frac{47(1.013^{\overbrace{25}^{3}} - 1)}{1.013 - 1}} \; 9$$

$$= \underbrace{1377.95}_{3} \text{ DOLLARS}$$

Consider the expression $(7x^{11} - 3x^5)^{21}$. 2 EACH EXCEPT AS NOTED **SCORE: _____ / 25 PTS**

[a] Write the first 3 terms of the expansion of the expression. Simplify all exponents.

Your answer may use multiplication and exponents, but NOT division, ! nor $_nC_r$ (or equivalent) notation.

$$\underbrace{(7x^{11})^{21}}_{} + \underbrace{\binom{21}{1}(7x^{11})^{20}(-3x^5)}_{} + \underbrace{\binom{21}{2}(7x^{11})^{19}(-3x^5)^2}_{}$$

$$= 7^{21}x^{231} - 21\cdot 7^{20}\cdot 3x^{225} + \boxed{\frac{21!}{2!19!}}7^{19}3^2x^{219}$$

$$\boxed{\frac{21\cdot \overset{10}{\cancel{20}}\cdot 19!}{\cancel{2}\cdot 1\cdot 19!}}$$

$$= \underbrace{7^{21}x^{231}}_{1} - \underbrace{21}_{1}\cdot\underbrace{7^{20}\cdot 3x^{225}}_{1\frac{1}{2}} + \underbrace{21\cdot 10\cdot 7^{19}\cdot 3^2}_{1\frac{1}{2}}\underbrace{x^{219}}_{1\frac{1}{2}}$$

[b] Find the coefficient of x^{159} in the expansion.

Your answer may use multiplication, division, exponents and !, but NOT $_nC_r$ (or equivalent) notation.

$$\underbrace{\binom{21}{r}(7x^{11})^{21-r}(-3x^5)^r}_{} = \binom{21}{r}7^{21-r}(-3)^r x^{11(21-r)+5r}$$

$$\underbrace{11(21-r)+5r=159}_{}$$

$$231-6r=159$$

$$-6r=-72$$

$$\underbrace{r=12}_{}$$

$$\text{COEFFICIENT} = \underbrace{\binom{21}{12}7^9(-3)^{12}}_{1\frac{1}{2}} = \underbrace{\frac{21!}{12!9!}7^9 3^{12}}_{}$$

Prove that $\sum_{i=1}^{n}[(i+1)\cdot(i+1)!-1]=(n+2)!-n-2$ for all positive integers n **using mathematical induction**. **SCORE: _____ / 25 PTS**

BASIS CASE: $\sum_{i=1}^{1}[(i+1)\cdot(i+1)!-1] = 2\cdot 2!-1 = 3 \quad 3!-1-2 = 6-1-2 = 3$

INDUCTIVE STEP: ASSUME $\sum_{i=1}^{k}[(i+1)\cdot(i+1)!-1] = (k+2)!-k-2$ FOR SOME ARBITRARY INTEGER $k \geq 1$

$$\sum_{i=1}^{k+1}[(i+1)\cdot(i+1)!-1]$$

$$= \sum_{i=1}^{k}[(i+1)\cdot(i+1)!-1] + (k+2)\cdot(k+2)!-1$$

$$= (k+2)!-k-2 + (k+2)\cdot(k+2)!-1$$

$$= (1+k+2)\cdot(k+2)!-k-3$$

1 $$= (k+3)\cdot(k+2)!-(k+1)-2$$

$$= (k+3)!-(k+1)-2$$

$$= ((k+1)+2)!-(k+1)-2$$

2 EACH EXCEPT AS NOTED

SO, BY MI,

1 $\sum_{i=1}^{n}[(i+1)(i+1)!-1] = (n+2)!-n-2$ FOR ALL POSITIVE INTEGERS n 1

Simplify $\begin{pmatrix} 3n-2 \\ 3n-5 \end{pmatrix}$.

SCORE: _____ / 15 PTS

$$= \boxed{\frac{(3n-2)!}{(3n-5)!\,3!}} = \boxed{\frac{(3n-2)\overset{n-1}{\cancel{(3n-3)}}(3n-4)\cancel{(3n-5)!}}{\cancel{(3n-5)!}\,\cancel{3}\cdot 2\cdot 1}}$$

5 5

$$= \underbrace{\tfrac{1}{2}(3n-2)(n-1)(3n-4)}_{5}$$

Find a_n for the geometric sequence with $a_3 = -\frac{12}{x^2 y}$ and $a_6 = \frac{3y^{11}}{2x^8}$.

SCORE: _____ / 20 PTS

$$a_n = a_1 r^{n-1}$$

$$a_3 = a_1 r^2 = \frac{-12}{x^2 y}$$

$$a_6 = a_1 r^5 = \frac{3y^{11}}{2x^8}$$

$$\frac{a_1 r^5}{a_1 r^2} = \frac{\frac{3y^{11}}{2x^8}}{\frac{-12}{x^2 y}}$$ 3

$$r^3 = -\frac{1}{8}\frac{y^{12}}{x^6}$$ 4

$$r = -\frac{y^4}{2x^2}$$ 3

$$a_1 \left(-\frac{y^4}{2x^2}\right)^2 = \frac{-12}{x^2 y}$$ 3

$$a_1 = -\frac{12}{x^2 y}\left(-\frac{2x^2}{y^4}\right)^2$$

$$= -\frac{12}{x^2 y}\frac{4x^4}{y^8}$$

$$= -\frac{48x^2}{y^9}$$ 4

$$a_n = -\frac{48x^2}{y^9}\left(-\frac{y^4}{2x^2}\right)^{n-1}$$ 3

Use sigma notation to write the series $-9+25-49+81-\cdots-961$.

SCORE: _____ / 15 PTS

$$-3^2+5^2-7^2+9^2-\cdots-31^2$$

$$3, 5, 7, 9, \ldots, 31$$

ARITHMETIC SEQUENCE

$$a_n = \underbrace{3+2(n-1)=31}_{2}$$

$$2(n-1)=28$$

$$n-1=14$$

$$\underbrace{n=15}_{2}$$

$$3\ \boxed{\sum_{n=1}^{15} \underbrace{(-1)^n}_{2} \left(\underbrace{3+2(n-1)}_{3}\right)^{\overset{3}{\overbrace{2}}}}$$

$$= \sum_{n=1}^{15} (-1)^n (2n+1)^2$$